Math 20550 - Summer 2016
Moments, Cylindrical and Spherical Coordinates
July 6, 2016

Problem 1. *Find the center of mass of the lamina described by*

$$0 \leq x \leq 3, 0 \leq y \leq 9 - x^2$$

with density function $\rho(x, y) = xy$

Problem 2. *Find the center of mass of region bounded by* $x^2 + y^2 = 9$ *whose density is proportional to the square of the distance from the origin.*

We can also compute moments of inertia. For a point mass m, the moment of inertia about an axis is given by $I = md^2$, where d is the distance from the particle to the axis. So in the 2 dimensional case, the *moment of inertia* of a lamina D about the x axis is given by

$$I_x = \iint_D y^2 \rho(x, y) dA$$

and about the y axis is

$$I_y = \iint_D x^2 \rho(x, y) dA.$$

Problem 3. *Find the moments of inertia for the lamina in Problem 2.*

Problem 4. *Find the center of mass of the solid bounded by* $x = 0$, $x = 4$, $z = 4 - y^2$, *and* $z = 0$ *if the density is:*

(a) $\rho(x, y, z) = 1$ *(Hint: you should be able to figure out* $\bar{x}$ *and* $\bar{y}$ *without computing any integrals.)*

(b) $\rho(x, y, z) = x$. *First make a conjecture about how the center of mass changes from part (a), then compute the center of mass.*

Problem 5. *Compute the center of mass of the solid region bounded by* $x^2 + y^2 = 4$, $z = 0$, *and* $z = 4$, *which has density function* $\rho(x, y, z) = x^2$.

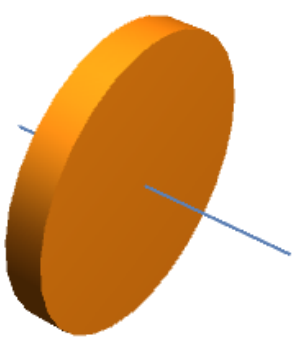
Problem 6. *Compute the triple integral*

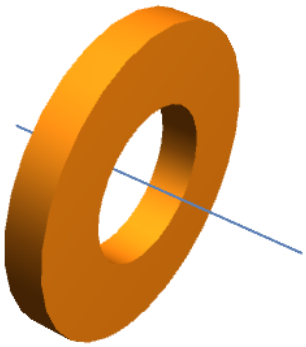
$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{4} x \; dz dy dx.$$

Problem 7. *Compute the triple integral*

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} dz dy dx.$$

Problem 8. *Consider the two solids* of equal weight *shown together with their axes of rotation*

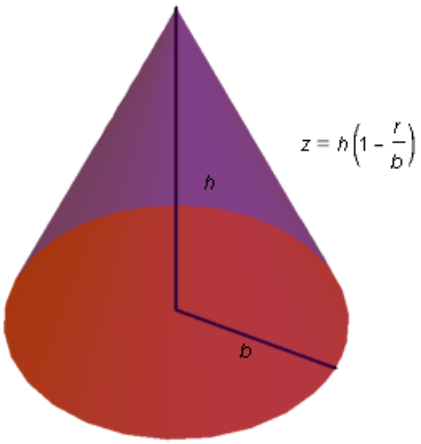




The radius of the disk is the same as the outer radius of the annulus. Assume the density is constant in both solids.

(a) *How do the centers of mass of the two solids differ?*
(b) *Because the solids have the same weight, which has the greater density? Why?*
(c) *Which solid has the greater moment of inertia?*
(d) *The solids are rolled down an inclined plane. They are started at the same time and at the same height. Which one reaches the bottom first? Why?*

Problem 9.



(a) *Find the volume of the cone of height h and base radius b.*
(b) *Find the center of mass of the cone. (Assume it has uniform density.)*

Problem 10. *Find the volume of the solid bounded by the cylinder $x^2 + z^2 = 4$, the plane $y = 0$, and the paraboloid $y = 4 - x^2 - x^2$.*

Problem 11. *Find the volume of the solid bounded by the cone $z = \sqrt{3x^2 + 3y^2}$ and the sphere $x^2 + y^2 + z^2 = 25$.*